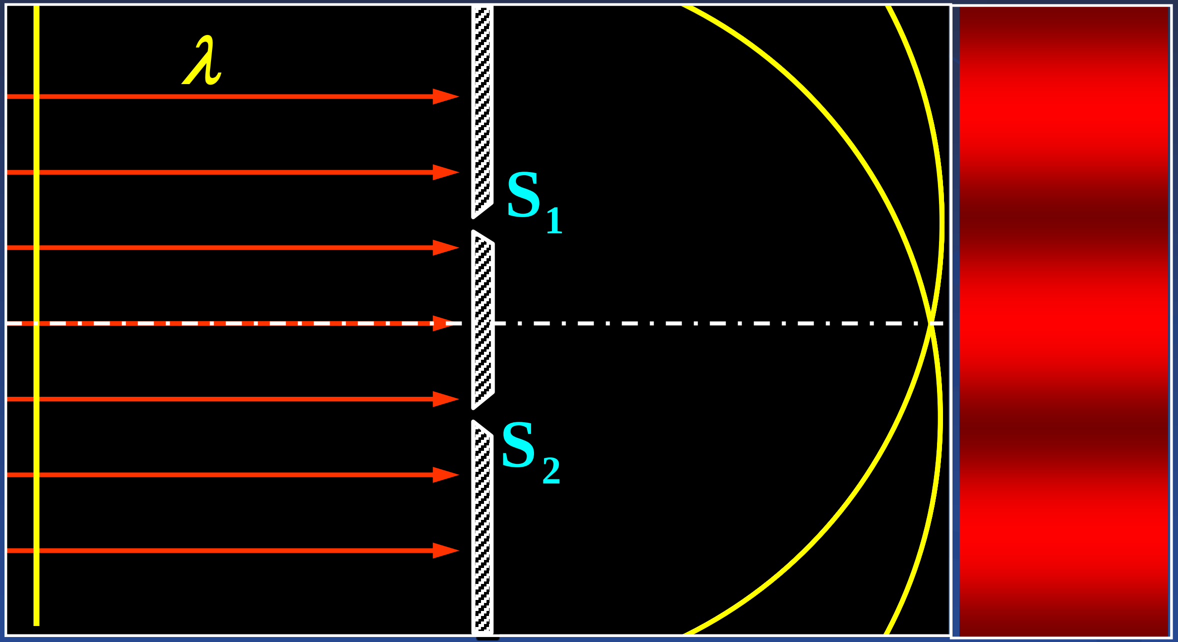
§ 11.2 杨氏双缝干涉

Thomas Youny

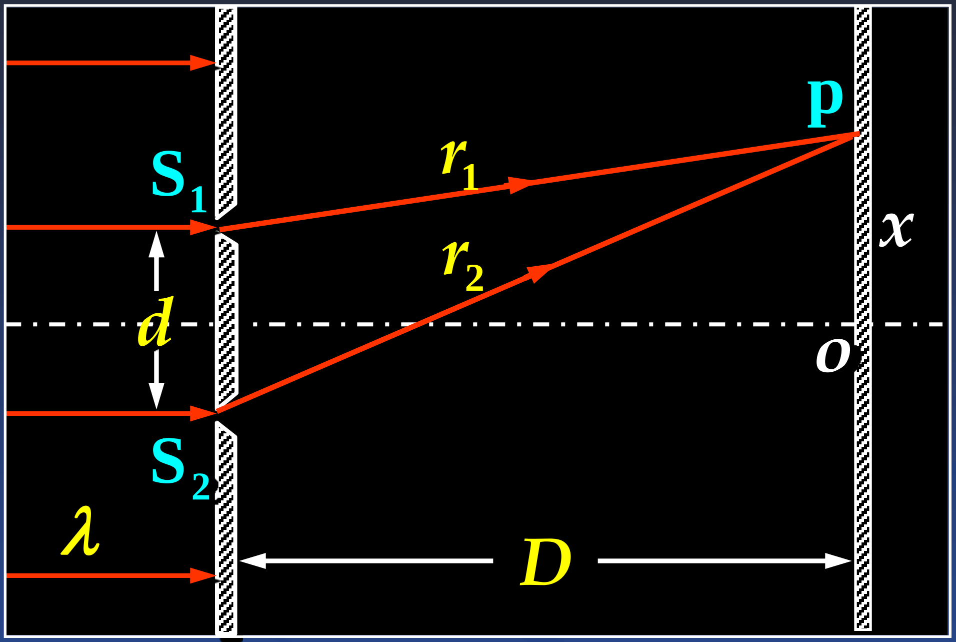
在1801年首先用实验的方法研究了光的干涉现象，为光的波动理论确定了实验基础。

实验装置图：



一、观测屏上的光强分布

p点光振动方程:



$$E_{1p} = A_{10}\cos(\omega t - \frac{2\pi}{\lambda} r_1)$$

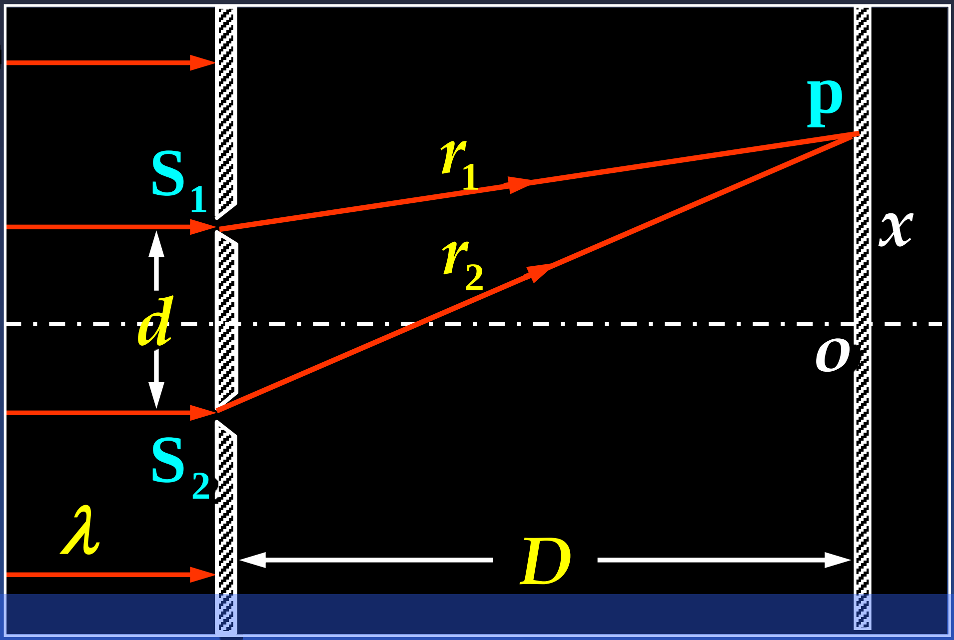
$$E_{2p} = A_{20}\cos(\omega t - \frac{2\pi}{\lambda} r_2)$$

$$E_p = A_p \cos(\omega t + \varphi)$$

$$A_p^2 = A_{10}^2 + A_{20}^2 + 2A_{10}A_{20}\cos(\Delta\phi) \qquad \Delta\phi = \frac{2\pi}{\lambda}(r_2 - r_1)$$

光强：$I_p \propto A_p^2$， $I_1 \propto A_{10}^2$， $I_2 \propto A_{20}^2$

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$$



设 $x \ll D$ （旁轴条件）

$\longrightarrow$ $I_1 \approx I_2 = I_0$

$$E_p = A_p \cos(\omega t + \varphi)$$

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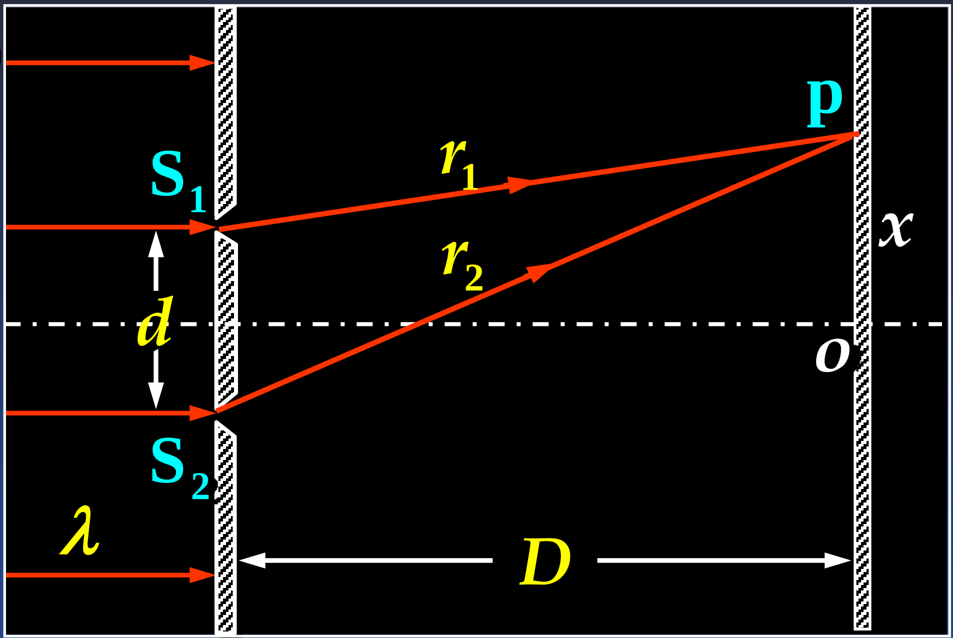

设 $x \ll D$ （旁轴条件）

$\longrightarrow$ $I_1 \approx I_2 = I_0$

$$\boxed{I_p = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)}$$

$$\Delta\phi = \frac{2\pi}{\lambda}(r_2 - r_1)$$

$I_{pmin} = 0 \le I_p \le I_{pmax} = 4I_0$

$\Delta\phi$与p点位置有关！

$$\Delta\phi = \begin{cases} \pm 2k\pi & I_p = I_{pmax} \\ \pm(2k+1)\pi & I_p = I_{pmin} \end{cases}$$

干涉加强 → 明纹

干涉减弱 → 暗纹

$(\ k = 0,\ 1,\ 2,\ \cdots)$

$$\Delta\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = \frac{2\pi}{\lambda}\delta$$

（δ 为光程差）

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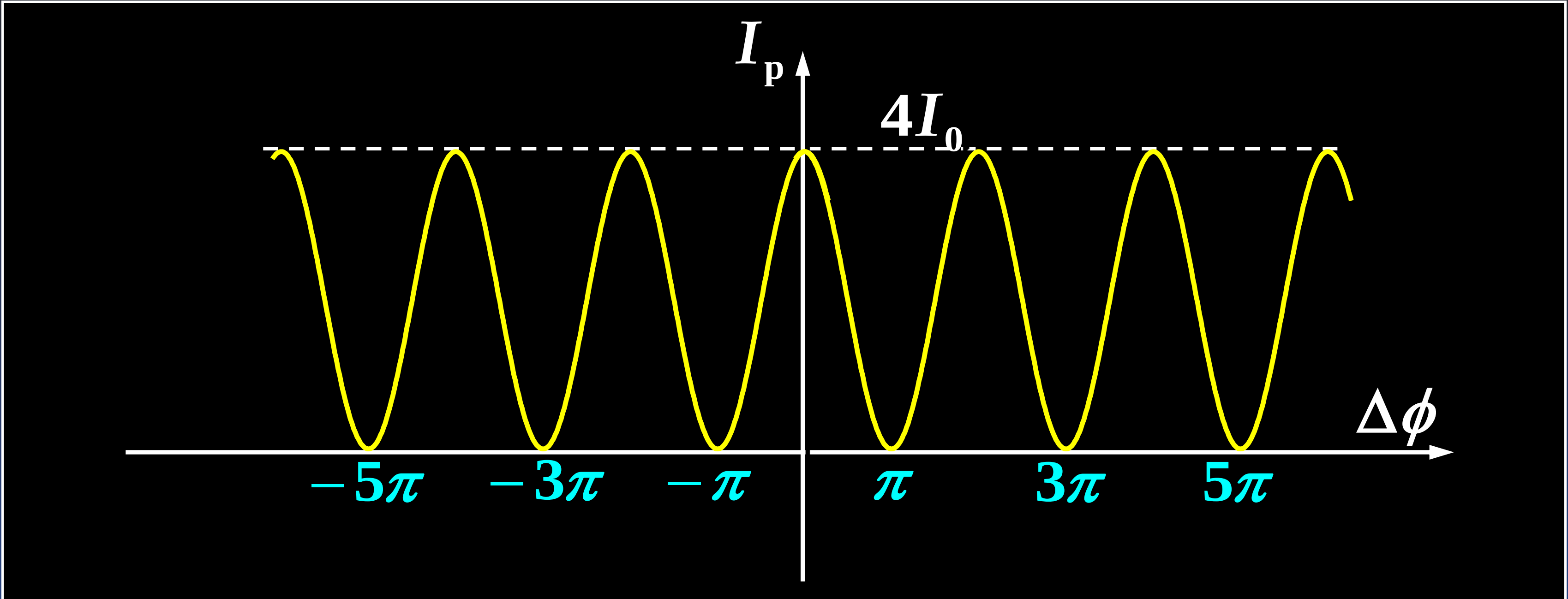
$$\delta = \begin{cases} \pm 2k\dfrac{\lambda}{2} & I_p = I_{pmax} \\ \pm(2k+1)\dfrac{\lambda}{2} & I_p = I_{pmin} \end{cases}$$

干涉加强 → 明纹

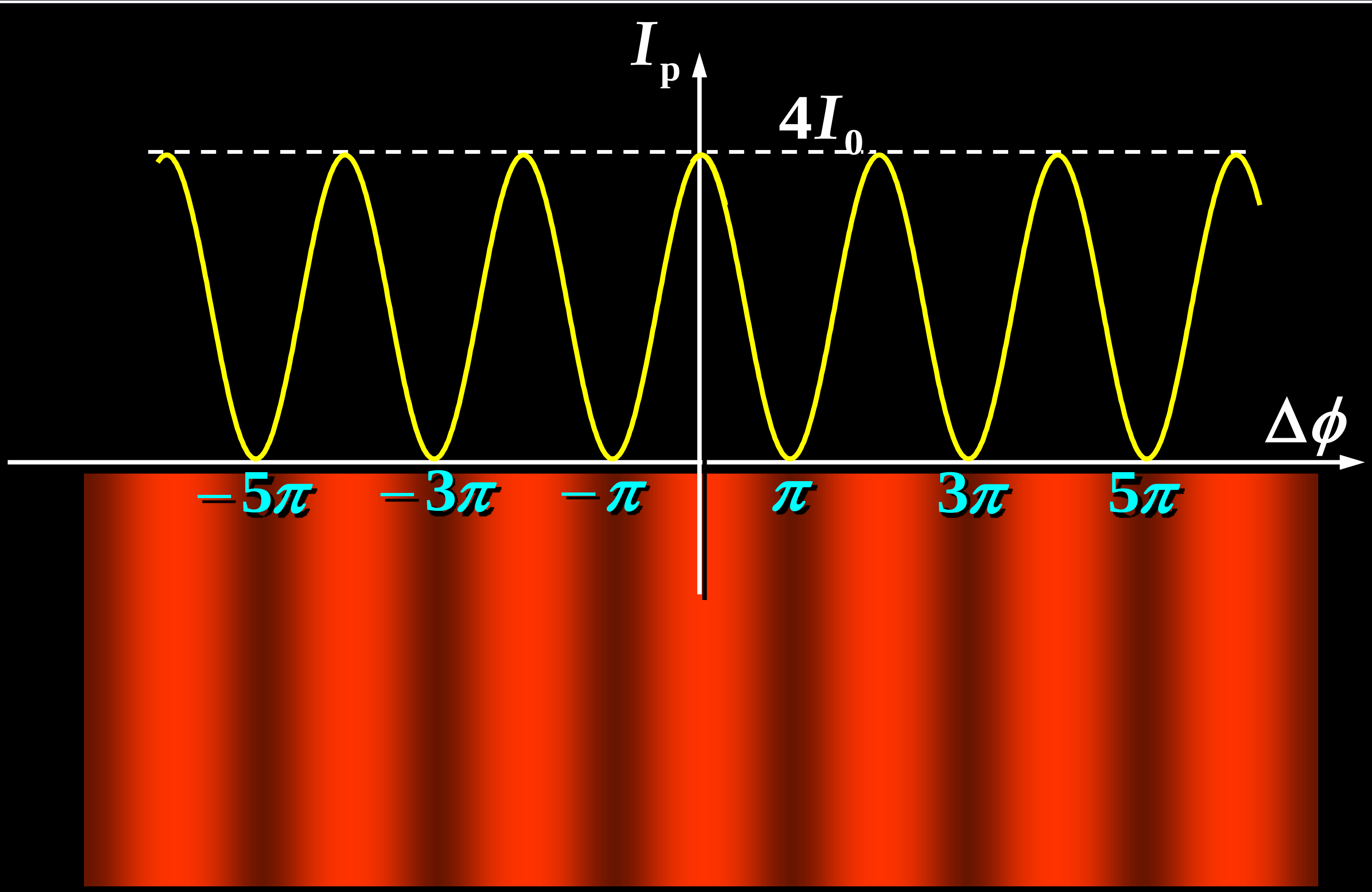
干涉减弱 → 暗纹

观测屏上光强分布曲线：



$$\delta=\begin{cases}\pm 2k\dfrac{\lambda}{2} & I_p=I_{pmax}\quad \text{干涉加强}\rightarrow\text{明纹}\\ \pm(2k+1)\dfrac{\lambda}{2} & I_p=I_{pmin}\quad \text{干涉减弱}\rightarrow\text{暗纹}\end{cases}$$

观测屏上光强分布曲线：



干涉图样：平行等间距的直条纹！

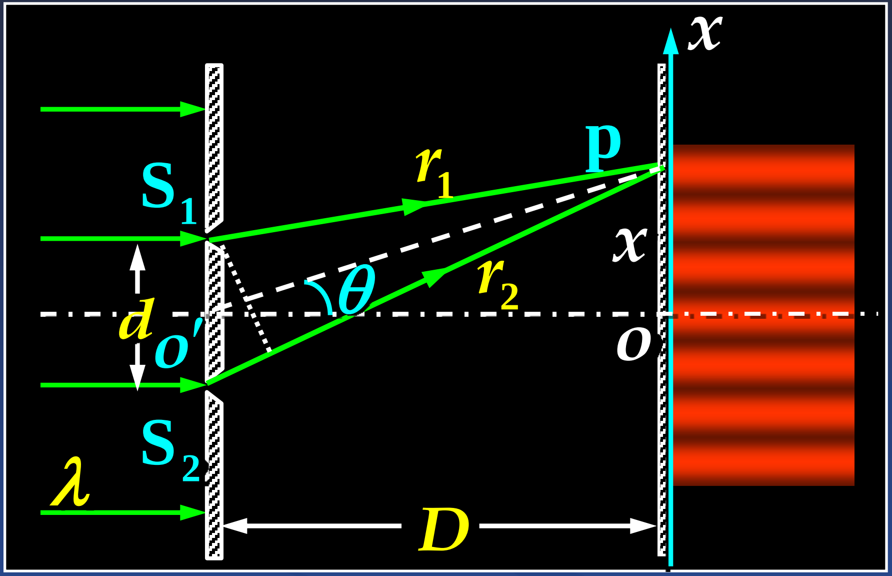
二、干涉条纹位置分布

设 入射光垂直入射， $d << D$ ， $x << D$

$$\delta = r_2 - r_1 \approx d \cdot \sin\theta$$

(θ 称为 衍射角)

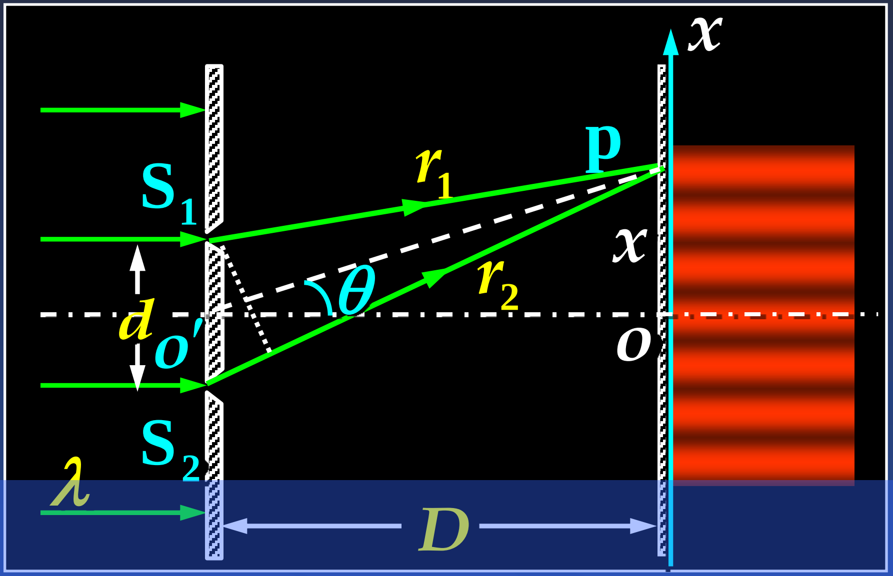
$$\sin\theta \approx tg\theta = \frac{x}{D}$$



$$\delta = r_2 - r_1 \approx d \cdot \frac{x}{D} = \begin{cases} \pm 2k\dfrac{\lambda}{2} & \text{明纹} \\ \pm(2k+1)\dfrac{\lambda}{2} & \text{暗纹} \end{cases}$$

明纹位置： $x_k = \pm \frac{D}{d} \cdot 2k \cdot \frac{\lambda}{2}$　　$(\ k = 0,\ 1,\ 2,\ \cdots)$

中央明纹： $k = 0,\ \delta = 0$



暗纹位置：

$$x_k = \pm \frac{D}{d} \cdot (2k+1) \cdot \frac{\lambda}{2}$$

$$\sin\theta \approx tg\theta = \frac{x}{D}$$

$$\delta = r_2 - r_1 \approx d \cdot \frac{x}{D} = \begin{cases} \pm 2k\dfrac{\lambda}{2} & \text{明纹} \\ \pm(2k+1)\dfrac{\lambda}{2} & \text{暗纹} \end{cases}$$

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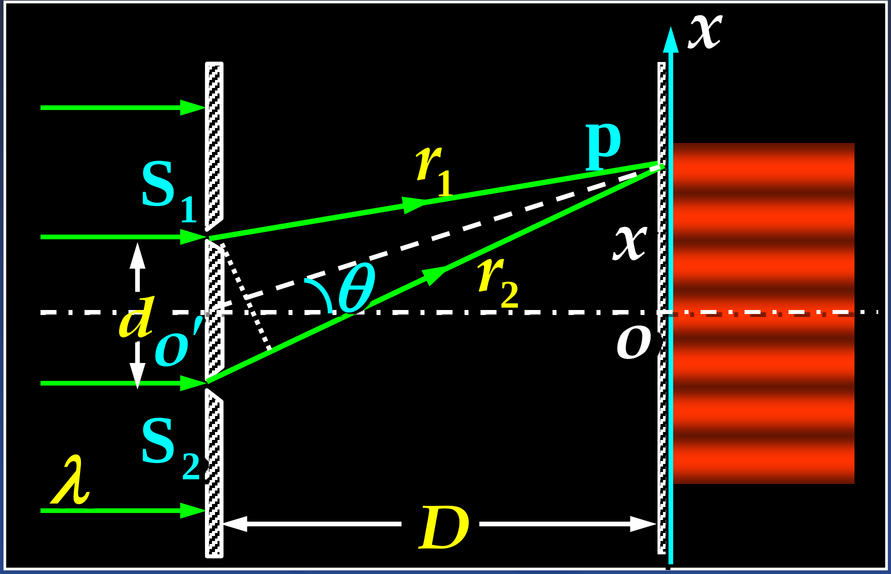
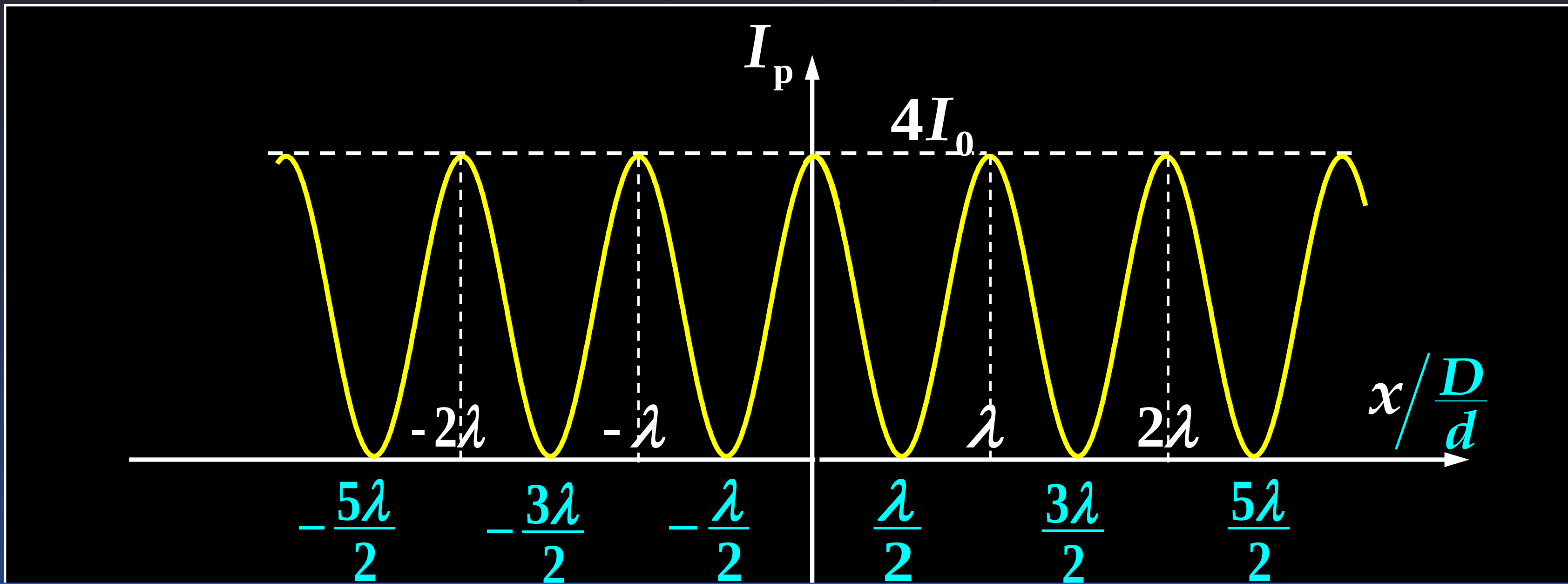


关于中央明纹对称！

相邻两条明(暗)纹间距：

$$\Delta x = \left| x_{k+1} - x_k \right| = \frac{D}{d}\lambda$$

观测屏上光强曲线(按位置分布):



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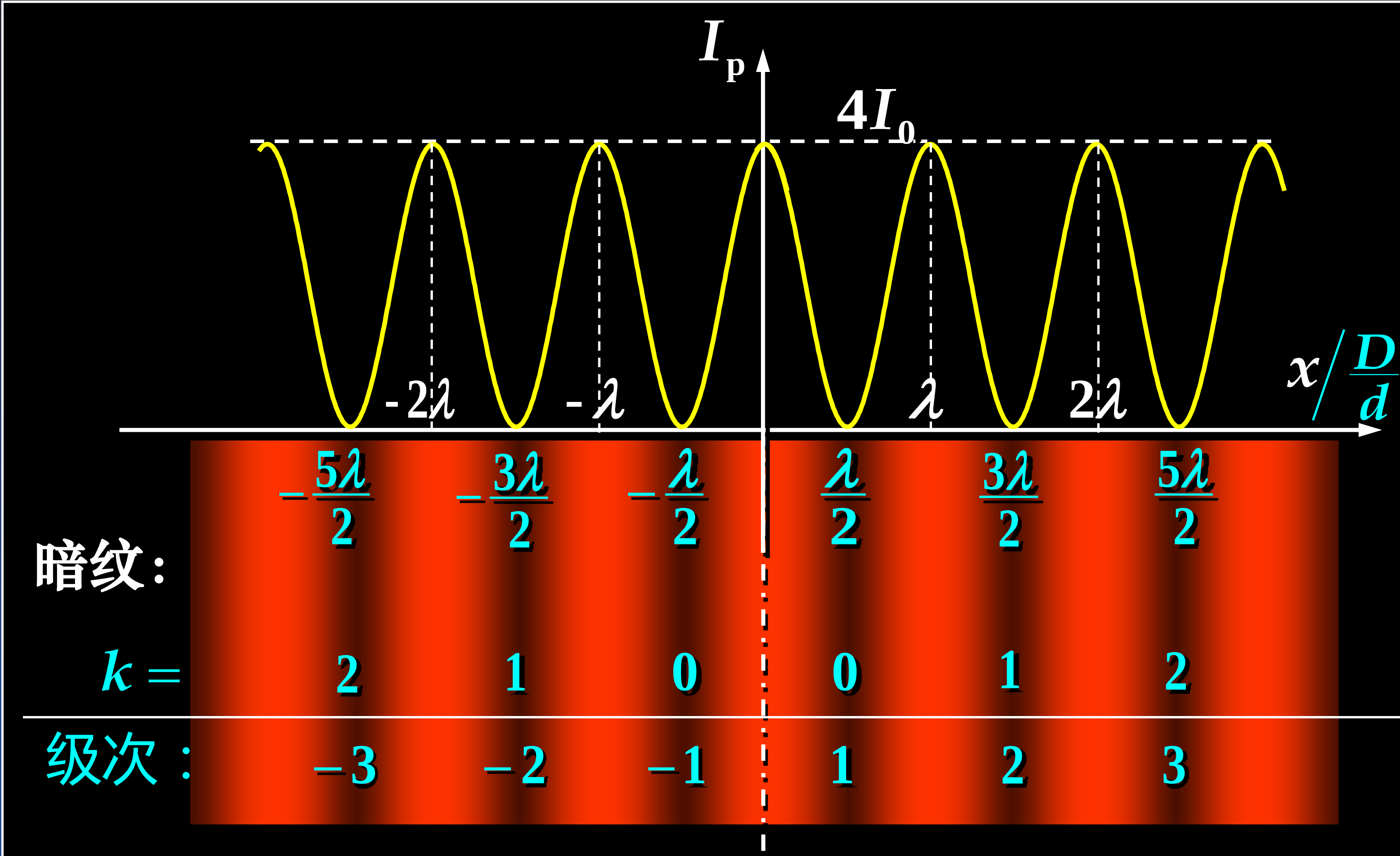

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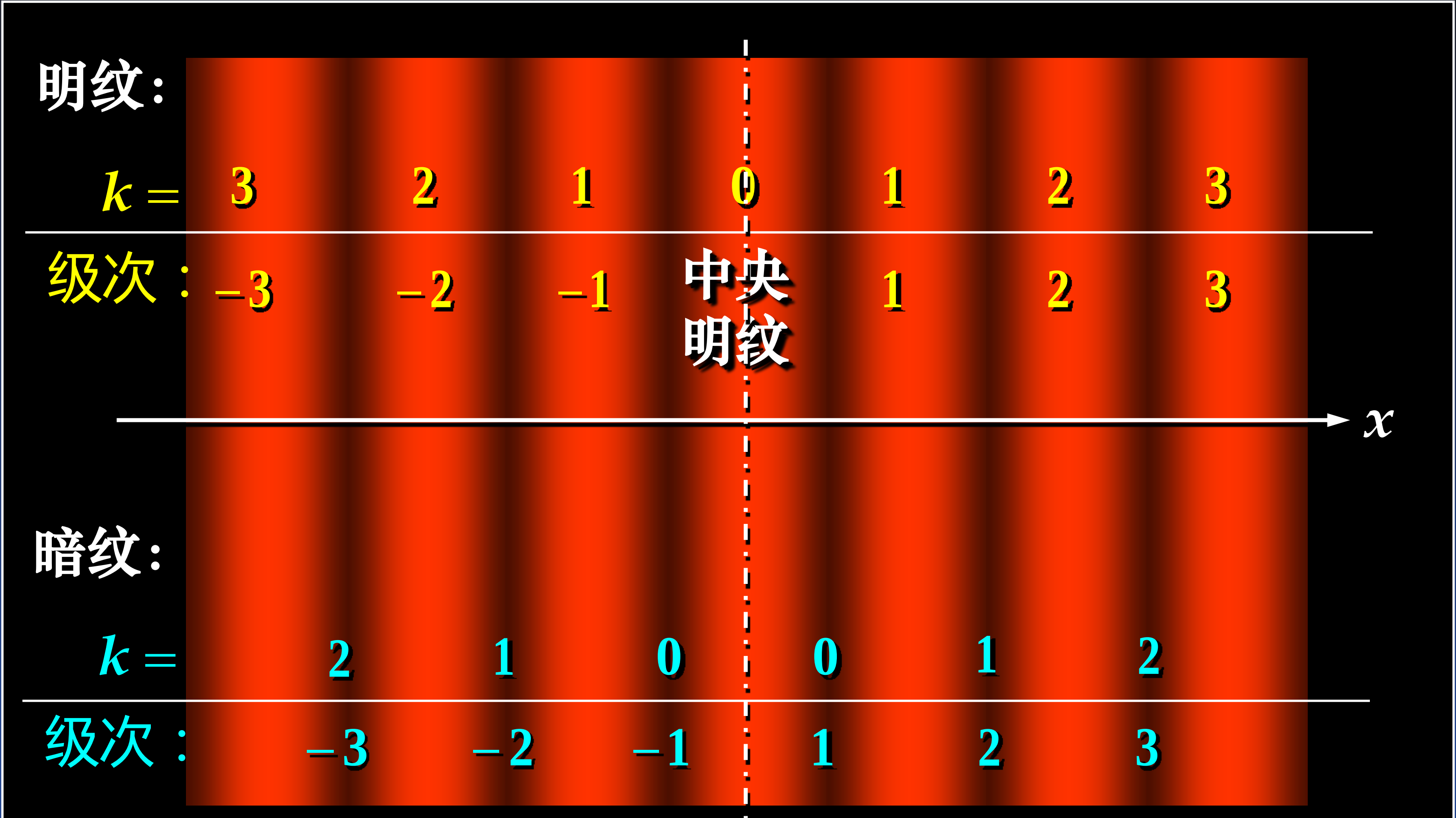
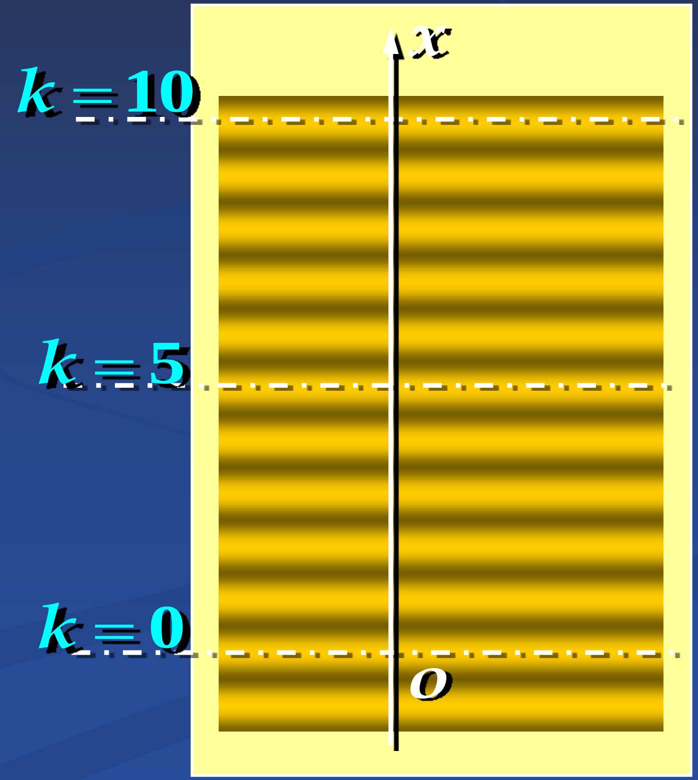


干涉图样：平行等间距的直条纹!

例 如图，$\lambda = 5500$Å垂直入射，$d = 2\times10^{-4}$ m， $D = 2$ m 求 (1) 第10级明纹间距；(2) 用厚度 $e = 6.7\times10^{-6}$ m、$n = 1.58$的云母片覆盖一缝后，中央明纹移到原来的某级明纹处，原来该处明纹的级次为多少?



解 求第10级明纹间距：

$$x_{10} = 10\cdot\frac{D}{d}\lambda$$

$$x_{-10} = -10\cdot\frac{D}{d}\lambda$$

$$x_{10} - x_{-10} = 20\frac{D}{d}\lambda = 110\ (\text{mm})$$

例 如图，$\lambda = 5500\text{Å}$垂直入射，$d = 2\times10^{-4}$ m， $D = 2$ m

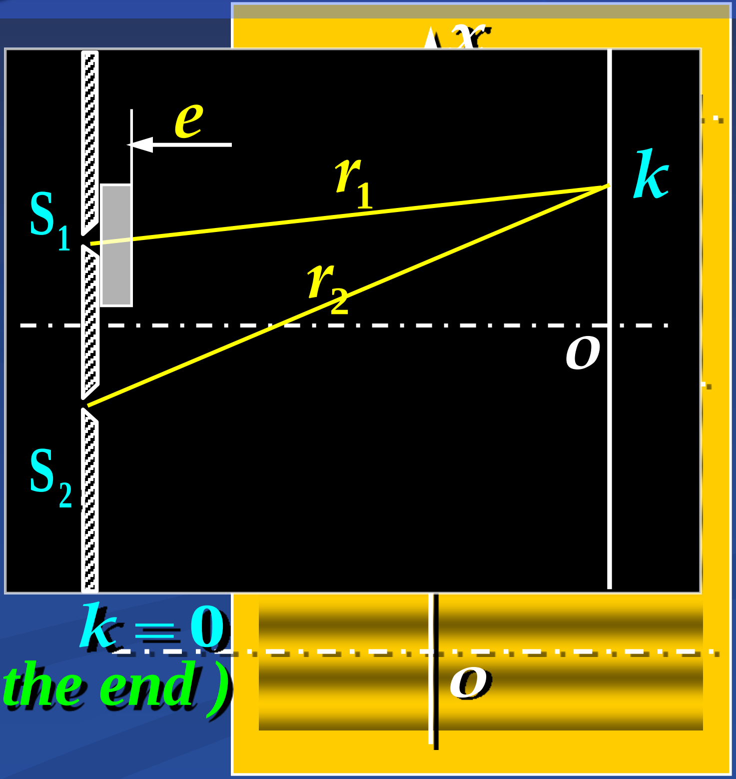
求 (1) 第10级明纹间距；(2) 用厚度 $e = 6.7\times10^{-6}$ m、$n = 1.58$的云母片覆盖一缝后，中央明纹移到原来的某级明纹处，原来该处明纹的级次为多少?

前: $\delta = r_2 - r_1 = 2k\dfrac{\lambda}{2}$

后: $\delta' = r_2 - (r_1 - e + ne) = 0$

$k = \dfrac{n-1}{\lambda}e \approx 7$

即移到原来的第 7 级明纹上。

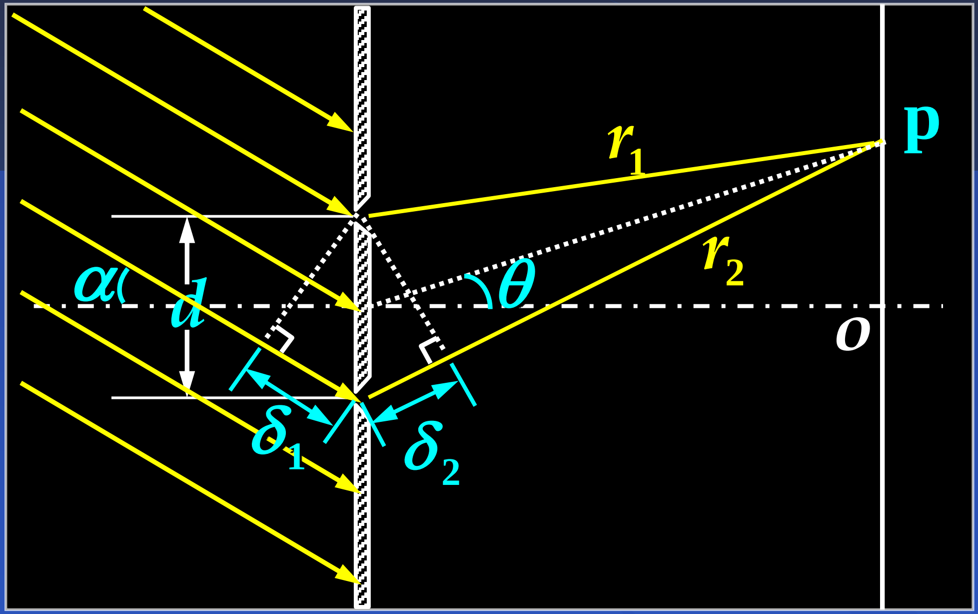


(the end)

三、几点讨论

1、斜入射： $\delta = \delta_1 + \delta_2 = d \cdot \sin\theta + d \cdot \sin\alpha$

$$\delta = d \cdot (\sin\theta + \sin\alpha)$$



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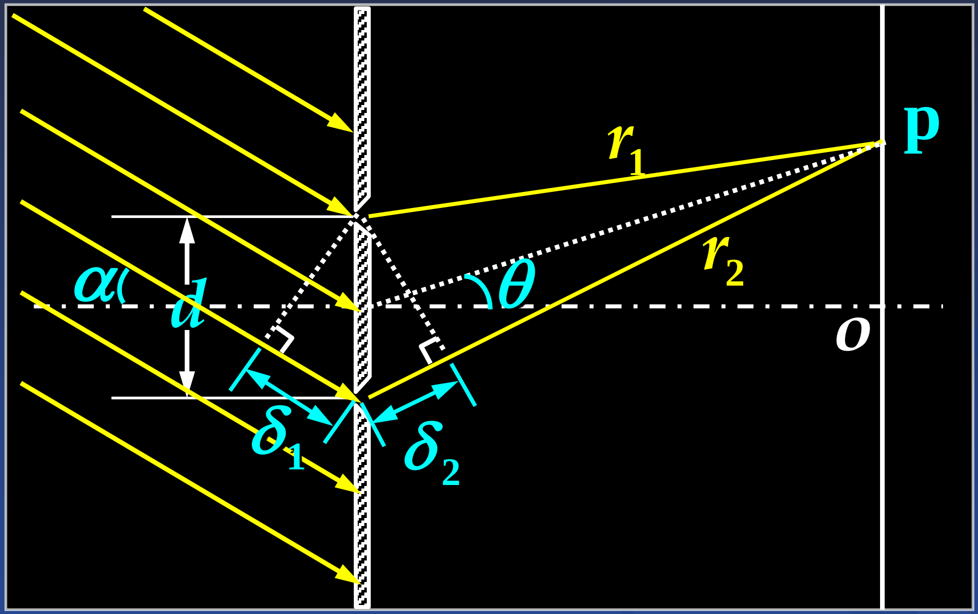
$$= \begin{cases} \pm 2k \cdot \lambda/2 & \text{明纹} \\ \pm (2k+1) \cdot \lambda/2 & \text{暗纹} \end{cases}$$

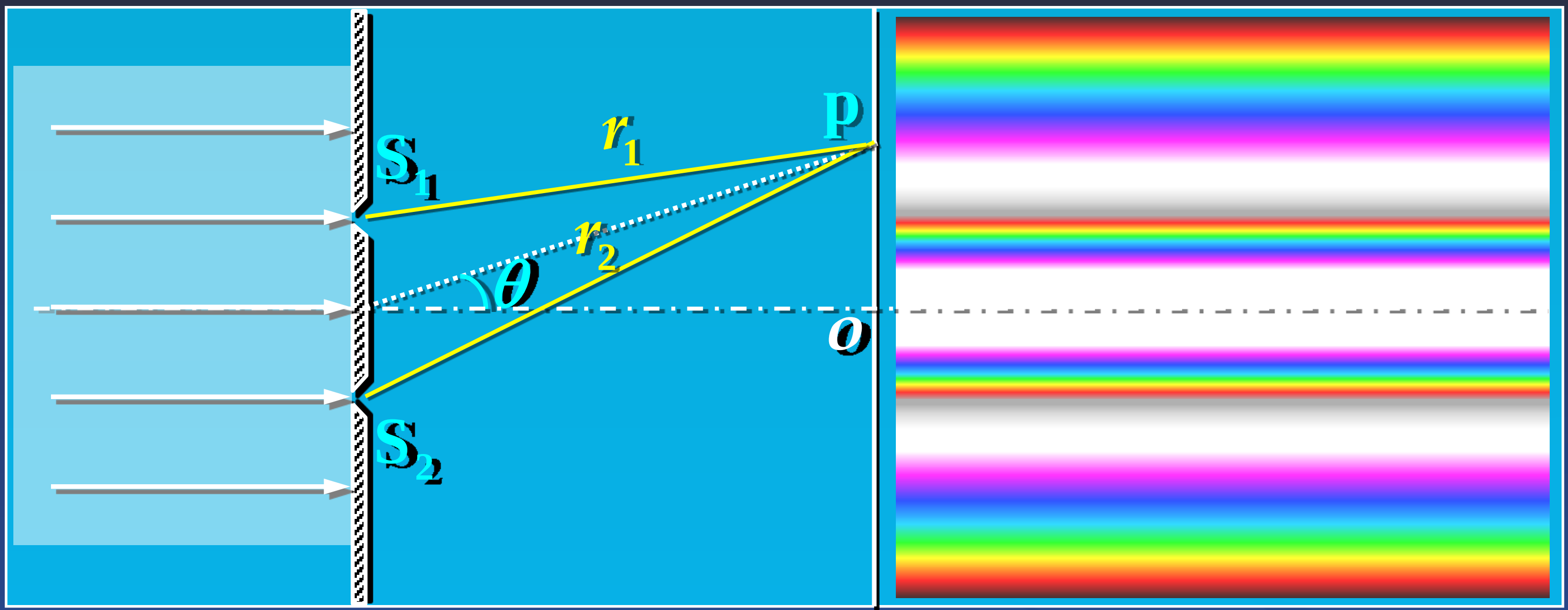
（注意 θ、α 的正负）



中央明纹出现在：$\delta = d \cdot (\sin\theta + \sin\alpha) = 0 \rightarrow \theta = -\alpha$

2、白光入射:

相邻条纹间距: $\Delta x = \dfrac{D}{d}\lambda \propto \lambda$



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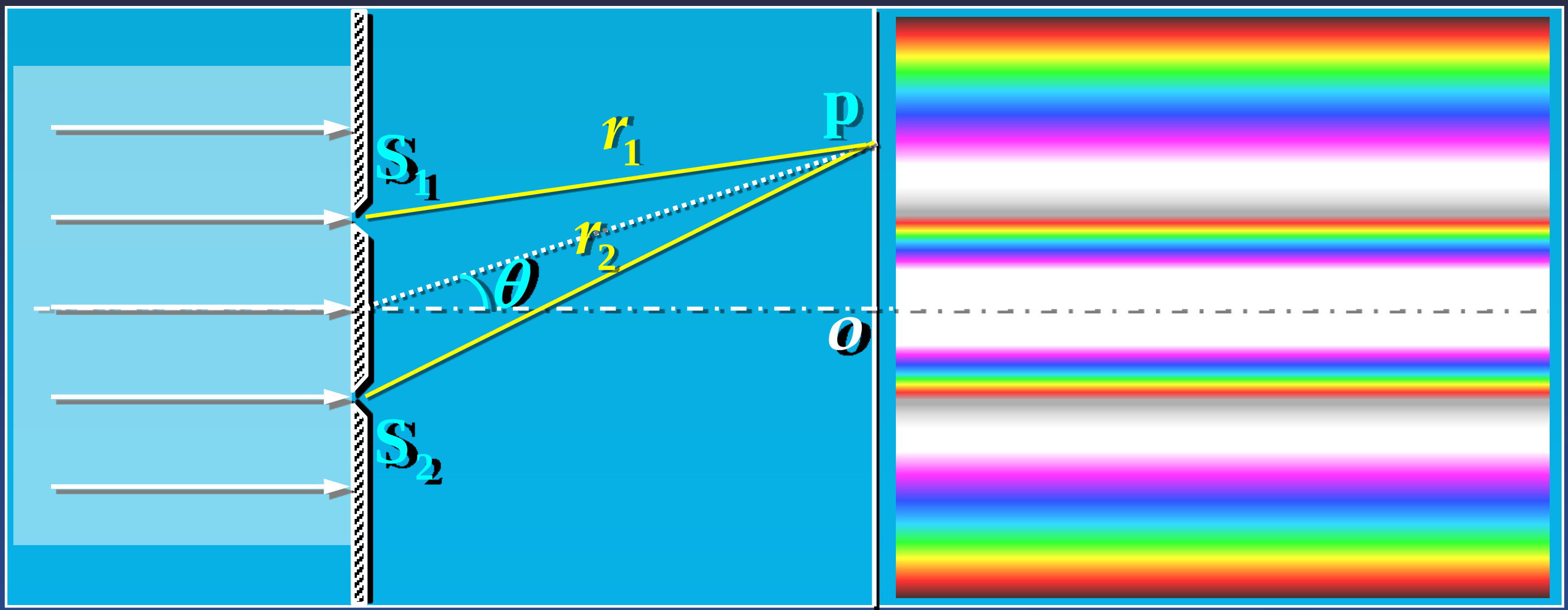
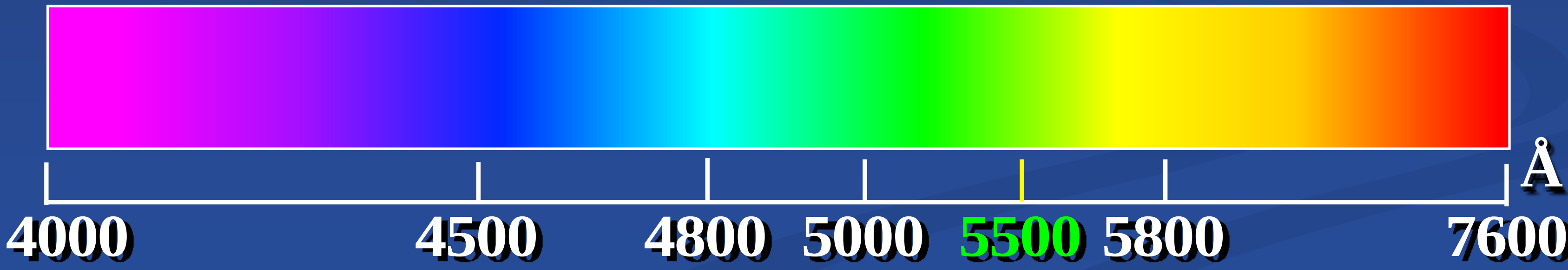
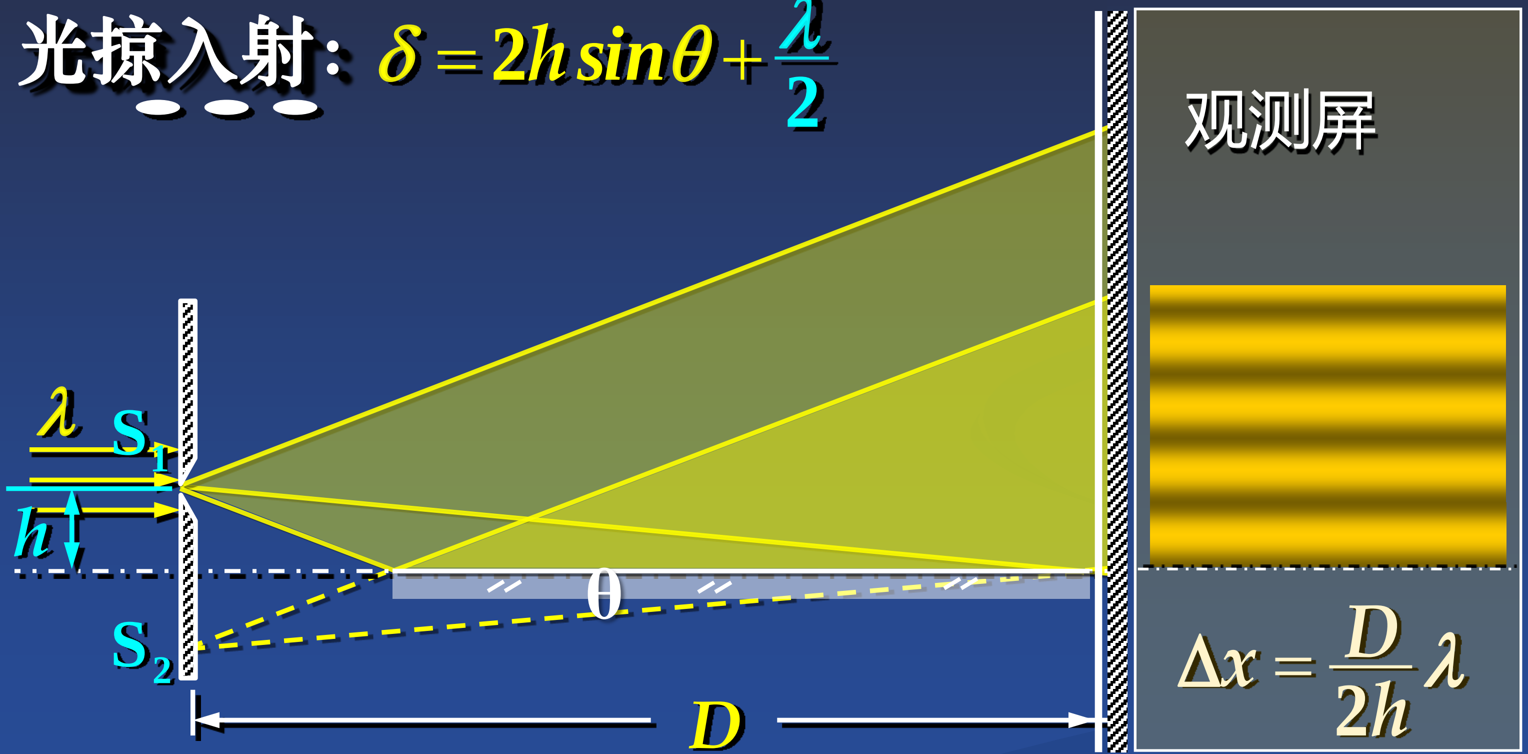




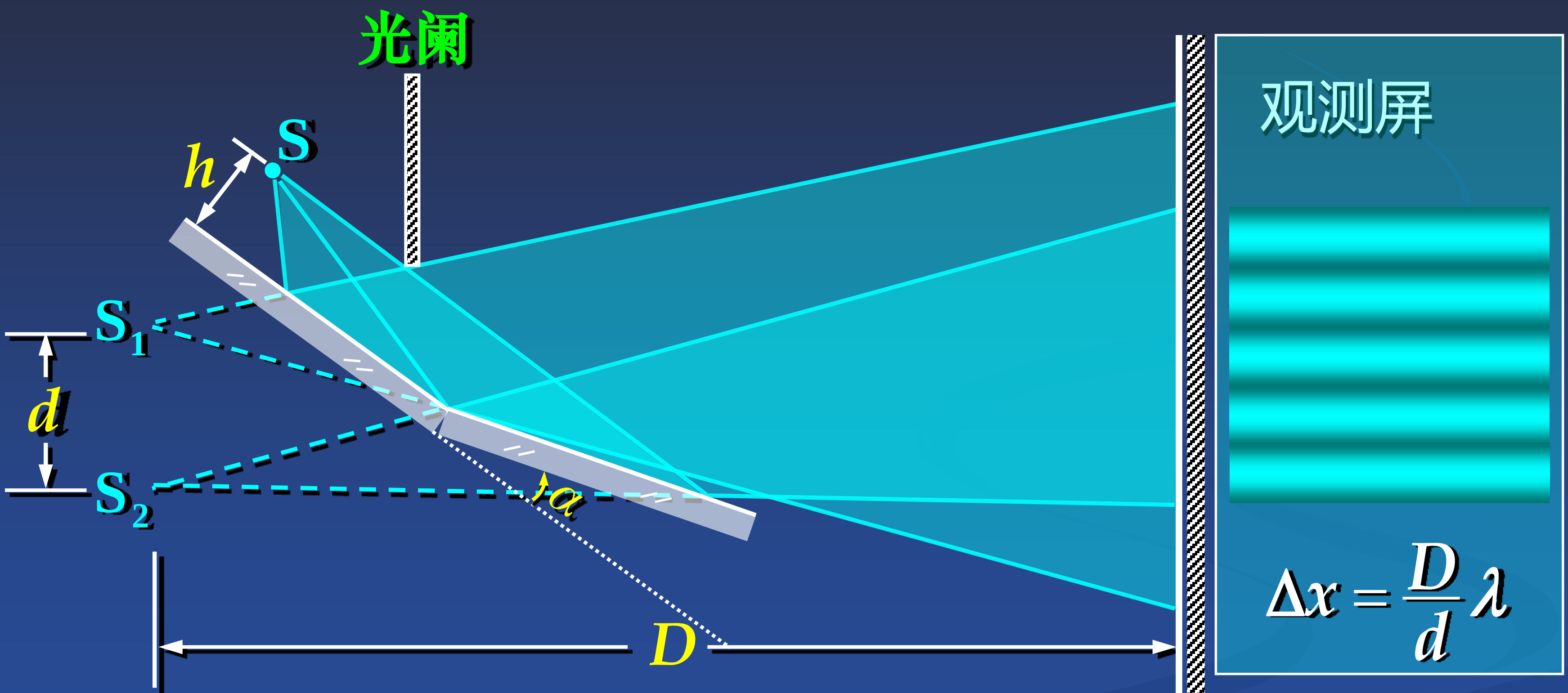
四、另外几种分波阵面法干涉实验

1、洛埃镜

光掠入射： $\delta = 2h\sin\theta + \dfrac{\lambda}{2}$



2、菲涅尔双镜



归纳:

1. 干涉规律:

$$\delta = r_2 - r_1 = \begin{cases} \pm 2k\dfrac{\lambda}{2} & \text{明纹} \\ \pm(2k+1)\dfrac{\lambda}{2} & \text{暗纹} \end{cases} \quad (k = 0,\ 1,\ 2,\ \cdots)$$

2. 相邻明纹(暗纹)间距: $\Delta x = \dfrac{D}{d}\lambda$

3. 双缝干涉条纹特点 : 平行、等间距、等亮度的条纹!

4. 条纹位置分布:

end